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Foundation Engineering

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Bearing Capacity of Shallow Foundations

Ultimate Bearing Capacity

Introduction

To perform satisfactorily, shallow foundations must have two main characteristics:

- They have to be safe against overall shear failure in the soil that supports them.
- They cannot undergo excessive displacement, or settlement. (The term *excessive* is relative, because the degree of settlement allowed for a structure depends on several considerations.)

- The load per unit area of the foundation at which shear failure in soil occurs is called the ***ultimate bearing capacity***

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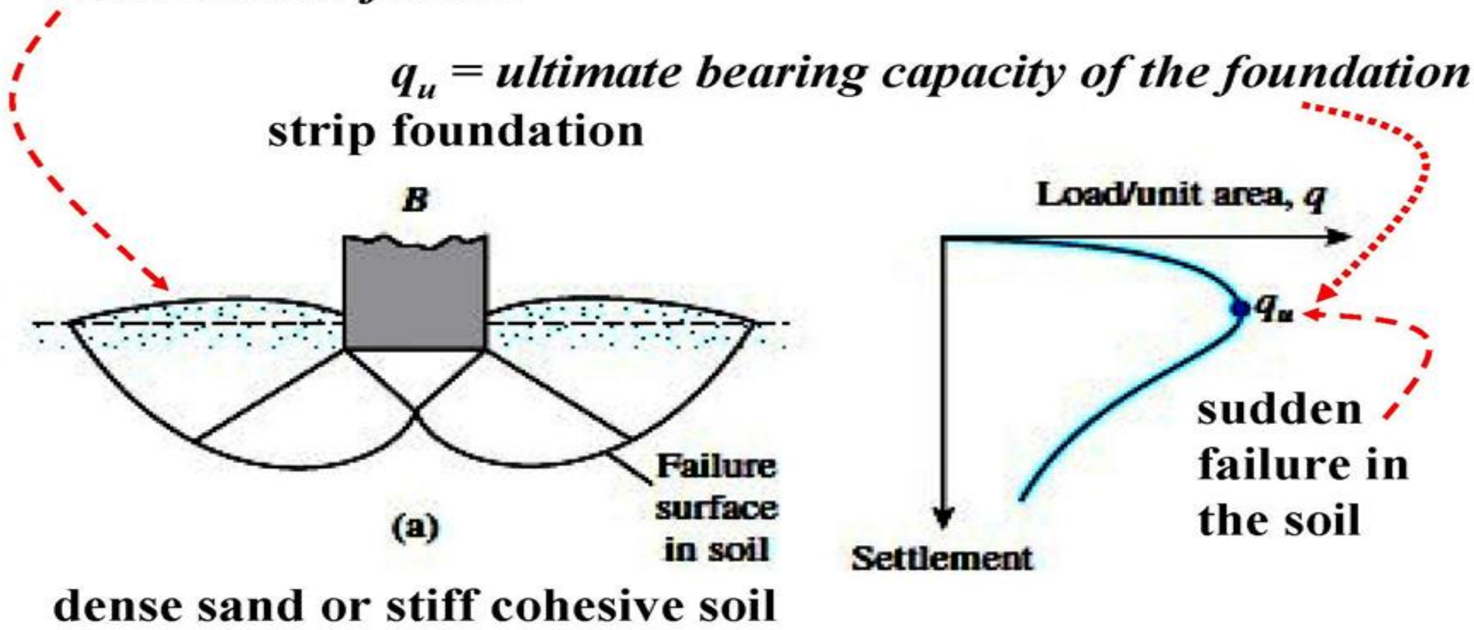
Ultimate Bearing Capacity

General Concept

General shear failure

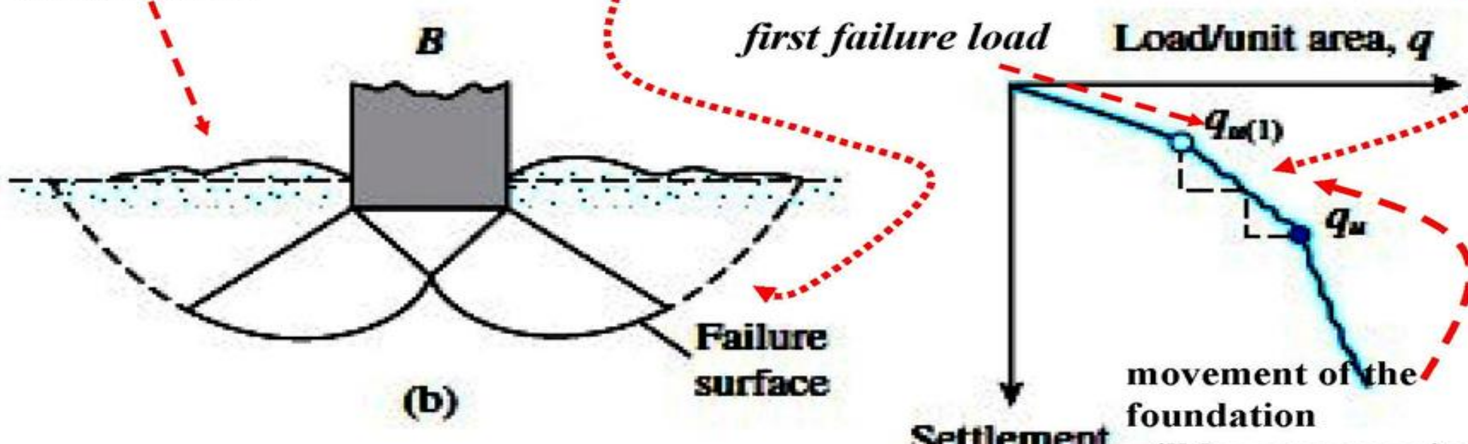
q_u = *ultimate bearing capacity of the foundation*



Ultimate Bearing Capacity

local shear failure **in soil.**

the failure surface in the soil will gradually extend outward from the foundation with a large increase of settlement



sand or clayey soil of medium compaction

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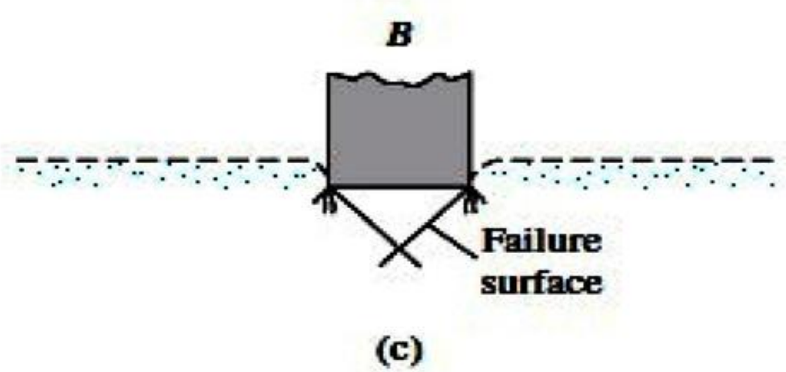
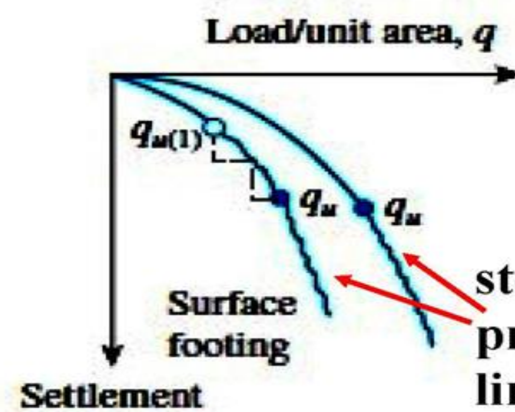
Punching shear failure.



fairly loose soil



steep and practically linear

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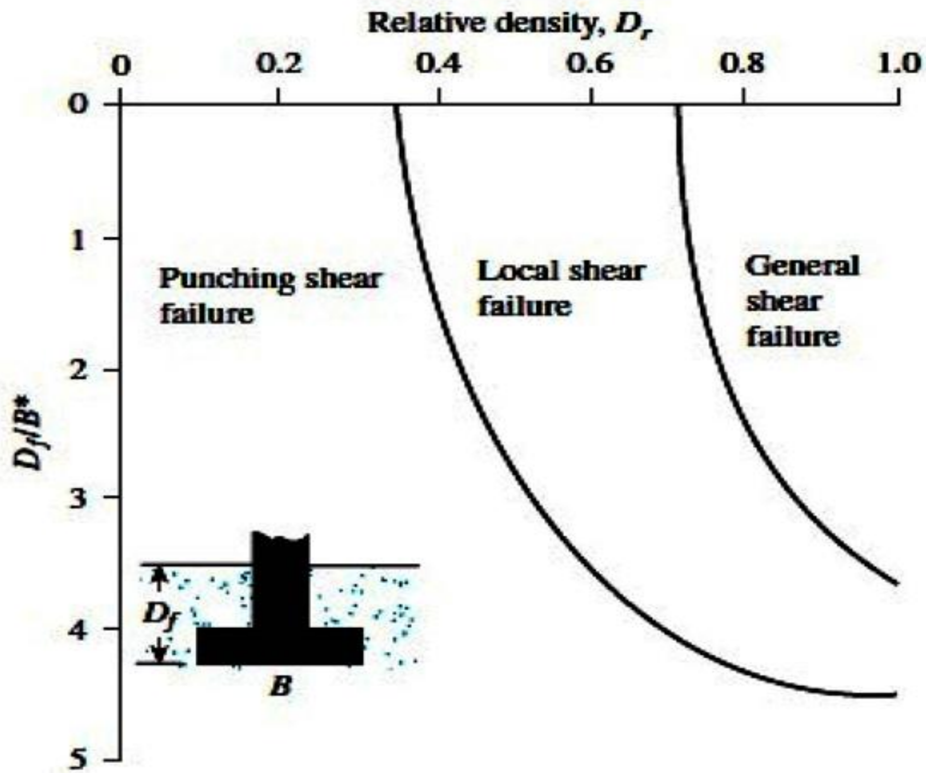
Modes of foundation failure in sand (Vesic, 1973)



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D_r = relative density of sand

D_f = depth of foundation measured from the ground surface

$$B^* = \frac{2BL}{B + L}$$

where

B = width of foundation
L = length of foundation

(*Note:* L is always greater than B.)

For square foundations, $B = L$;

for circular foundations, $B = L$ = diameter

$$B^* = B$$

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Ultimate Bearing Capacity

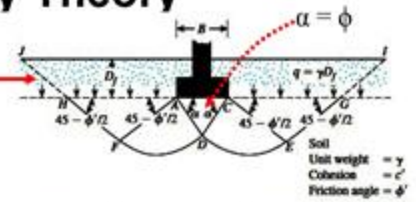
General Guidelines

- Footings in clays - *general shear*
- Footings in Dense sands ($D_r > 67\%$) -*general shear*
- Footings in Loose to Medium dense ($30\% < D_r < 67\%$) - *Local Shear*
- Footings in Very Loose Sand ($D_r < 30\%$)-*punching shear*

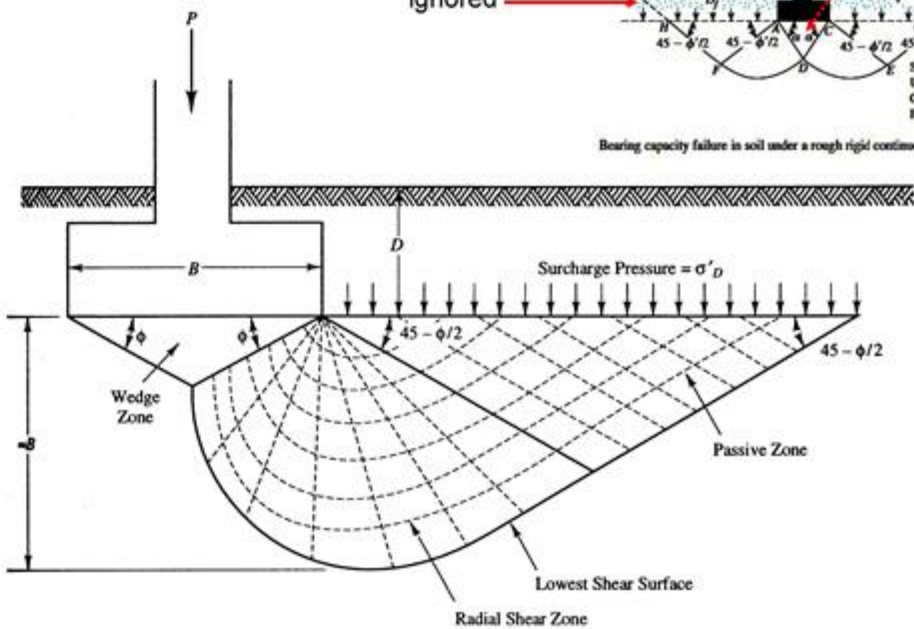
Ultimate Bearing Capacity

Terzaghi's Bearing Capacity Theory

ignored



Bearing capacity failure in soil under a rough rigid continuous (strip) foundation



Ultimate Bearing Capacity

Assumptions

- $D \leq B$
- No sliding between footing and soil
- soil: a homogeneous semi-infinite mass
- general shear failure
- footing is very rigid compared to soil

Ultimate Bearing Capacity

Terzaghi Bearing Capacity Formulas

For Continuous foundations:

$$q_{ult} = c'N_c + \sigma'_{zD}N_q + 0.5\gamma' BN_\gamma$$

For Square foundations:

$$q_{ult} = 1.3c'N_c + \sigma'_{zD}N_q + 0.4\gamma' BN_\gamma$$

For Circular foundations:

$$q_{ult} = 1.3c'N_c + \sigma'_{zD}N_q + 0.3\gamma' BN_\gamma$$

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Ultimate Bearing Capacity

$$N_c = \frac{N_q - 1}{\tan\phi'} \quad when \quad \phi' > 0$$

$$Nc = 5.7 \qquad when \quad \phi' = 0$$

$$N_q = \frac{a_\theta^2}{2\cos^2(45 + \phi'/2)}$$

$$a_\theta = \exp\left[\pi(0.75 - \phi'/360)\tan\phi'\right]$$

$$N_\gamma = \frac{\tan\phi'}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi'} - 1\right)$$

$\left.\right\}$ Terzaghi Bearing Capacity Factors

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Ultimate Bearing Capacity

For foundations that exhibit the local shear failure mode in soils, Terzaghi suggested the following modifications

$$q_u = \frac{2}{3}c'N'_c + qN'_q + \frac{1}{2}\gamma BN'_\gamma \qquad \text{(strip foundation)}$$

$$q_u = 0.867c'N'_c + qN'_q + 0.4\gamma BN'_\gamma \qquad \text{(square foundation)}$$

$$q_u = 0.867c'N'_c + qN'_q + 0.3\gamma BN'_\gamma \qquad \text{(circular foundation)}$$

N'_c, N'_q, and N'_γ, the ***modified bearing capacity factors***, can be calculated by using the bearing capacity factor equations (for N_c, N_q, and N_γ, respectively) by replacing ϕ' by $\overline{\phi}' = \tan^{-1}(\frac{2}{3}\tan\phi')$. The variation of N'_c, N'_q, and N'_γ with the soil friction angle ϕ' is given in Table Next page

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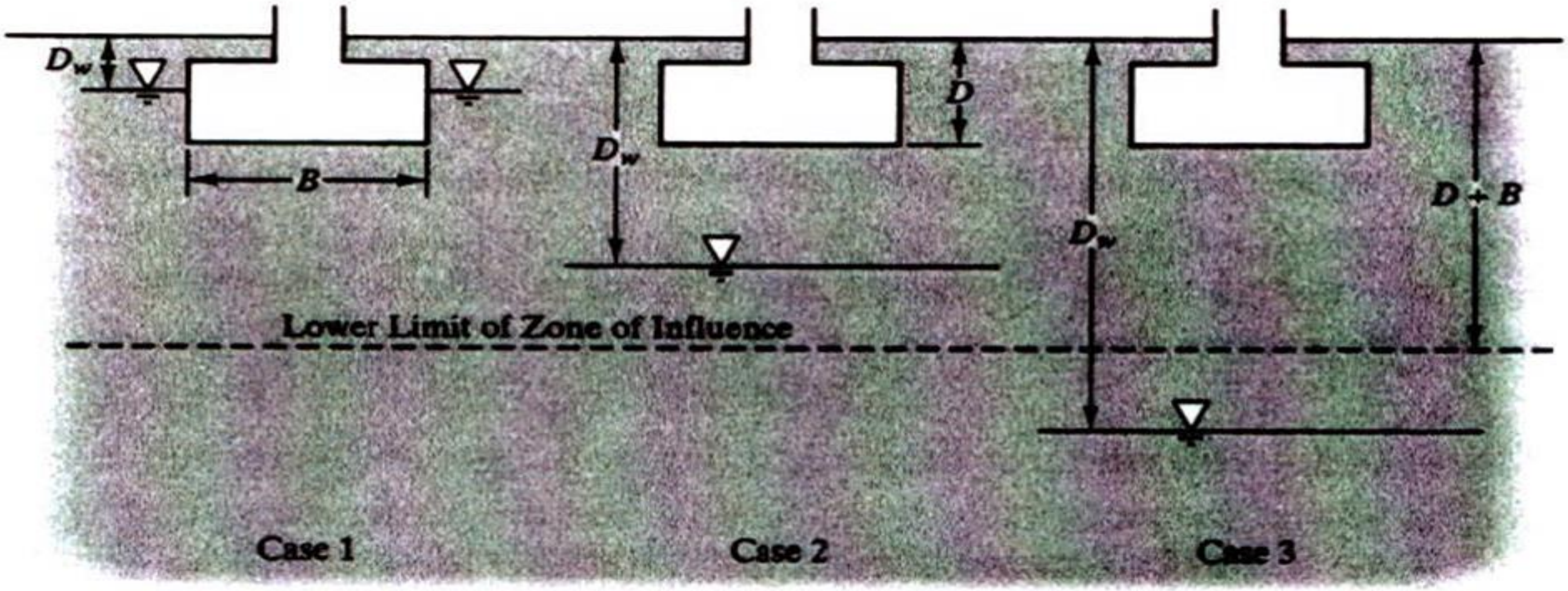
Ultimate Bearing Capacity

Groundwater Table Effect



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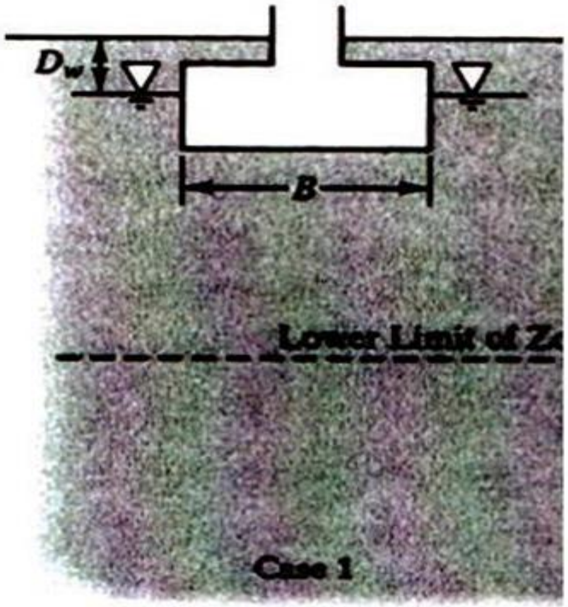
Ultimate Bearing Capacity



1. Modify σ'_{zD}
2. Calculate γ' as follows:

$$\gamma' = \gamma_b = \gamma - \gamma_w$$

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Ultimate Bearing Capacity

The General Bearing Capacity Equation

- The previous ultimate bearing capacity equations
  - do not address the case of rectangular foundations ($0<B/L<1$)
  - do not take into account the shearing resistance along the failure surface in soil above the bottom of the foundation
  - Do not take the load inclination on the foundation

- To account for all these shortcomings, Meyerhof (1963) suggested the following form of the general bearing capacity equation:

Ultimate Bearing Capacity

$$q_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

In this equation:

c' = cohesion
q = effective stress at the level of the bottom of the foundation
γ = unit weight of soil
B = width of foundation (= diameter for a circular foundation)

$$\left.\begin{aligned} F_{cs}, F_{qs}, F_{\gamma s} &= \text{shape factors} \\ F_{cd}, F_{qd}, F_{\gamma d} &= \text{depth factors} \\ F_{ci}, F_{qi}, F_{\gamma i} &= \text{load inclination factors} \end{aligned}\right\} \text{empirical factors}$$

N_c, N_q, N_γ = bearing capacity factors

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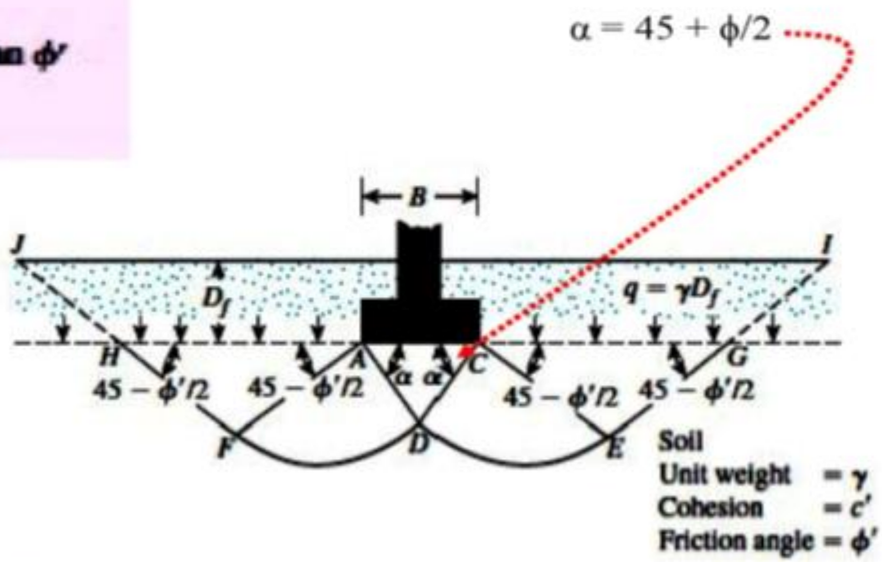
Ultimate Bearing Capacity

Bearing Capacity Factors

$$N_q = \tan^2\left(45 + \frac{\phi'}{2}\right)e^{\pi \tan \phi'}$$

$$N_c = (N_q - 1)\cot \phi'$$

$$N_\gamma = 2(N_q + 1)\tan \phi'$$



Bearing capacity failure in soil under a rough rigid continuous (strip) foundation

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Ultimate Bearing Capacity

- Shape, Depth, Inclination Factors

Factor	Relationship	Reference
Shape	$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$	DeBeer (1970)
	$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi'$	
	$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$	

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Depth

Hansen (1970)

$$\frac{D_f}{B} > 1$$

For $\phi = 0$:

$$F_{cd} = 1 + 0.4 \underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$$

$$F_{qd} = 1$$

$$F_{\gamma d} = 1$$

For $\phi' > 0$:

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi'}$$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$$

$$F_{\gamma d} = 1$$

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Ultimate Bearing Capacity

Depth $\frac{D_f}{B} \leq 1$ Hansen (1970)

For $\phi = 0$:

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$$

$$F_{qd} = 1$$
$$F_{\gamma d} = 1$$

For $\phi' > 0$:

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi'}$$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \left(\frac{D_f}{B}\right)$$

$$F_{\gamma d} = 1$$

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Inclination	$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2$	Meyerhof (1963); Hanna and Meyerhof (1981)
	$F_{\gamma i} = \left(1 - \frac{\beta}{\phi'}\right)$	
	β = inclination of the load on the foundation with respect to the vertical	

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Ultimate Bearing Capacity

Allowable Bearing Capacity

- gross *allowable load-bearing capacity*

$$q_{all} = \frac{q_u}{F}$$

F .... Factor of safety

- Net *allowable load-bearing capacity*

$$q_{all(net)} = \frac{q_u - q}{F}$$

$$q_u - q = q_{u(net)}$$

$$q = \gamma Df$$

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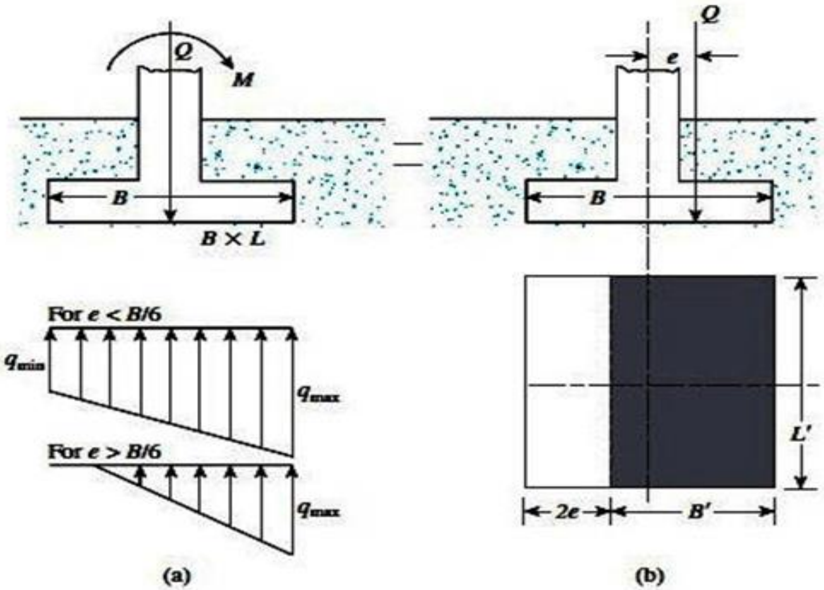
Ultimate Bearing Capacity

Eccentrically Loaded Foundations

- foundations may subjected to moments in addition to the vertical load, as shown in Figure.
- In such cases, the distribution of pressure by the foundation on the soil is not uniform.



Eccentrically loaded foundations

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Ultimate Bearing Capacity

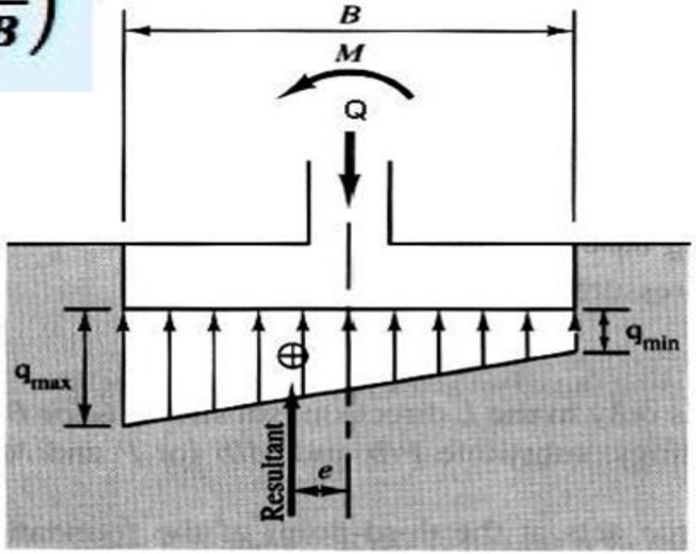
$$q_{max} = \frac{Q}{BL} + \frac{6M}{B^2L} \rightarrow q_{max} = \frac{Q}{BL}\left(1 + \frac{6e}{B}\right)$$

$$q_{min} = \frac{Q}{BL} - \frac{6M}{B^2L} \rightarrow q_{min} = \frac{Q}{BL}\left(1 - \frac{6e}{B}\right)$$

where

Q = total vertical load
M = moment on the foundation

$$e = \frac{M}{Q} \quad \text{eccentricity}$$



(a) $e < B/6$

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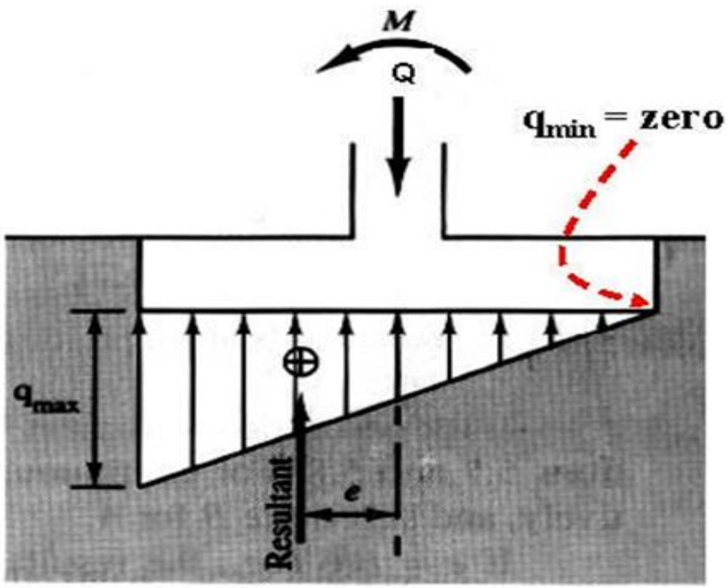
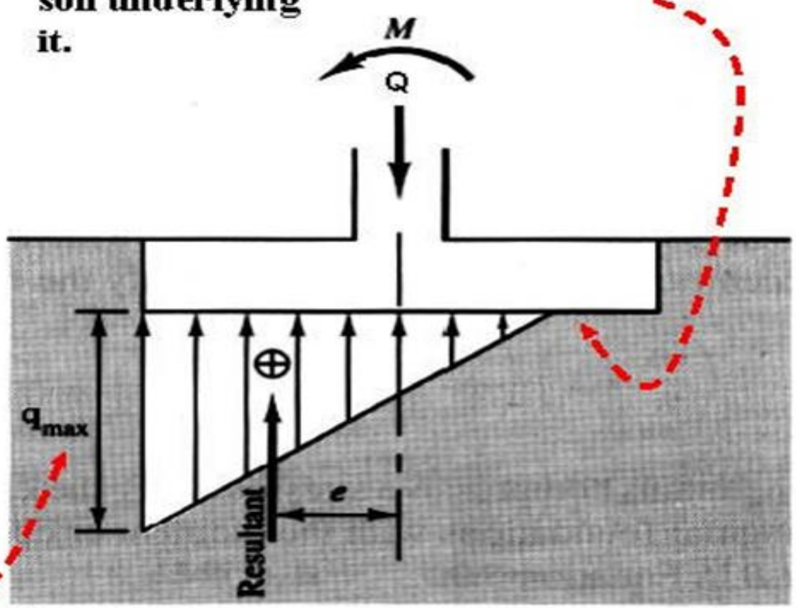
Ultimate Bearing Capacity

q_{min} will be negative, which means that tension will develop. Because soil cannot take any tension, there will then be a separation between the foundation and the soil underlying it.



(b) $e = B/6$

$$q_{max} = \frac{4Q}{3L(B - 2e)}$$



(c) $e > B/6$

Ultimate Bearing Capacity

Ultimate Bearing Capacity under Eccentric Loading—One Way Eccentricity

- **Effective Area Method (Meyerhoff, 1953)**

Step 1. Determine the effective dimensions of the foundation

$$B' = \text{effective width} = B - 2e$$
$$L' = \text{effective length} = L$$

Note that if the eccentricity were in the direction of the length of the foundation, the value of L' would be equal to $L - 2e$. The value of B' would equal B. The smaller of the two dimensions (i.e., L' and B') is the effective width of the foundation.

Step 2. the ultimate bearing capacity:

$$q'_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \tfrac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

To evaluate F_{cs}, F_{qs}, and $F_{\gamma s}$, with *effective length* and *effective width* dimensions instead of L and B, respectively. To determine F_{cd}, F_{qd}, and $F_{\gamma d}$, do not replace B with B'.

Step 3. The total ultimate load that the foundation can sustain is

$$Q_{ult} = q'_u \overbrace{(B')(L')}^{A'}$$

where A' = effective area.

Step 4. The factor of safety against bearing capacity failure is

$$FS = \frac{Q_{ult}}{Q}$$

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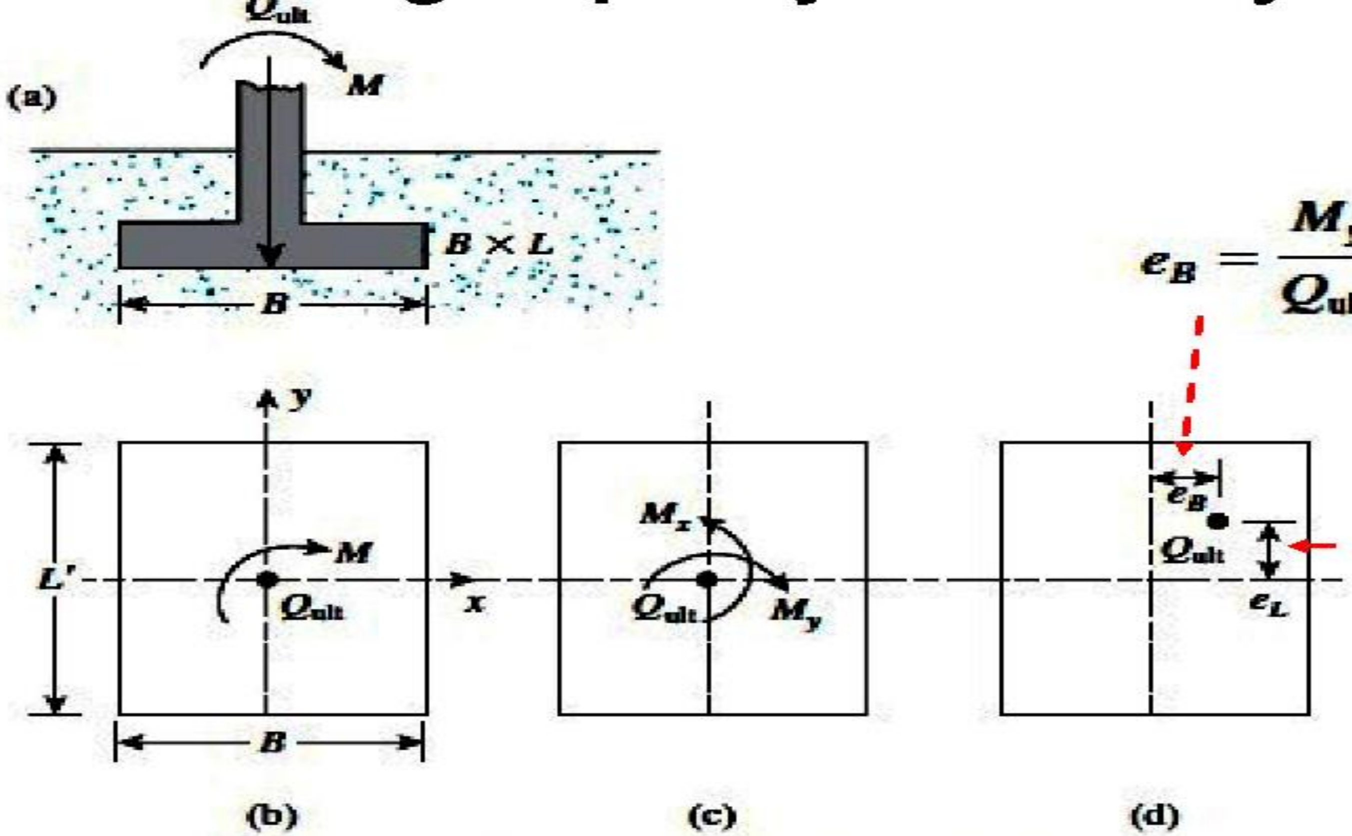
Ultimate Bearing Capacity

- **Bearing Capacity—Two-way Eccentricity**



$$e_B = \frac{M_y}{Q_{ult}}$$

$$e_L = \frac{M_x}{Q_{ult}}$$

Analysis of foundation with two-way eccentricity

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VTC

Ultimate Bearing Capacity

$$Q_{ult} = q'_u A'$$

where $q'_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$

and

$$A' = \text{effective area} = B' L'$$

As before, to evaluate F_{cs}, F_{qs}, and $F_{\gamma s}$ we use the effective length L' and effective width B' instead of L and B, respectively.

To calculate F_{cd}, F_{qd}, and $F_{\gamma d}$, we do not replace B with B'.

In determining the effective area A', effective width B', and effective length L', five possible cases may arise

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Ultimate Bearing Capacity

- Depending on loading conditions two way eccentricity is analyzed one of five ways.
  - 1. $e_L/L >= 1/6$ and $e_B/B >= 1/6$
  - 2. $e_L/L < 1/2$ and $e_B/B < 1/6$
  - 3. $e_L/L < 1/6$ and $e_B/B < 1/2$
  - 4. $e_L/L < 1/6$ and $e_B/B < 1/6$
  - 5. Circular footing – always 1 way

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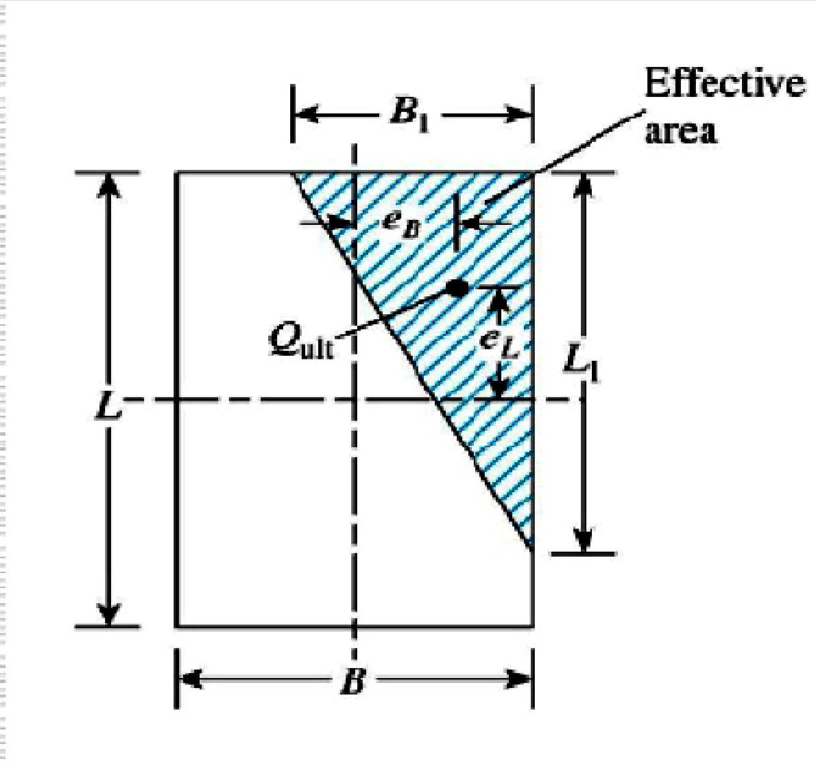
Ultimate Bearing Capacity

- *Case 1* $e_L/L >= 1/6$ and $e_B/B >= 1/6$



$$B_1 := B \cdot \left(1.5 - \frac{3\,e_B}{B}\right)$$

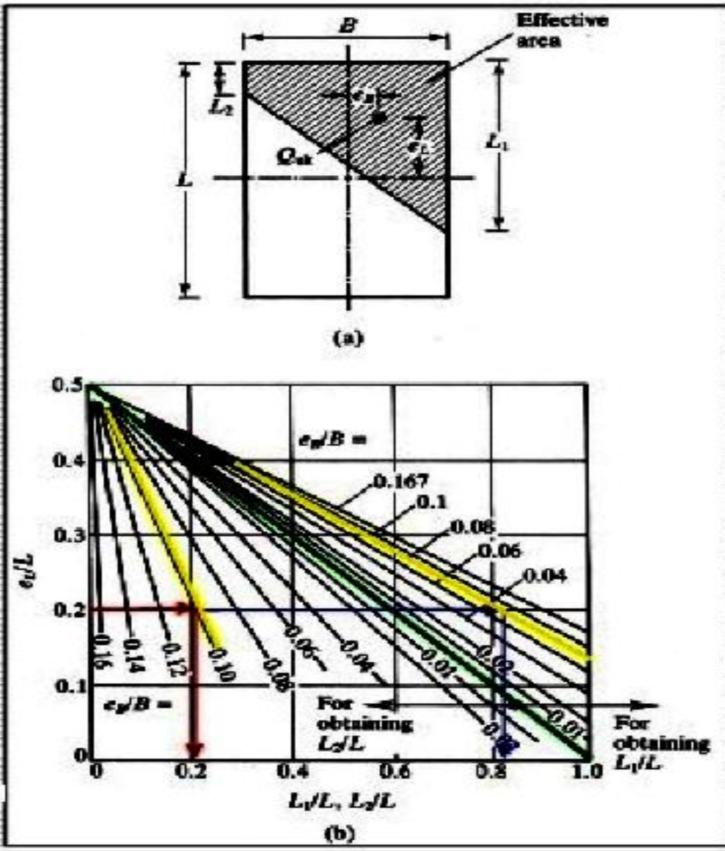
$$L_1 := L \cdot \left(1.5 - \frac{3\,e_L}{B}\right)$$

$$A' = \tfrac{1}{2} B_1 \bullet L_1$$

L' = larger of B_1 or L_1
SO $B' = A'/L'$

Ultimate Bearing Capacity

- Case 2 $e_L/L < 0.5$ and $0 < e_B/B < 1/6$



$$A' = \tfrac{1}{2}(L_1 + L_2)B$$

L' = larger of L_1 or L_2

$$B' = A'/L'$$

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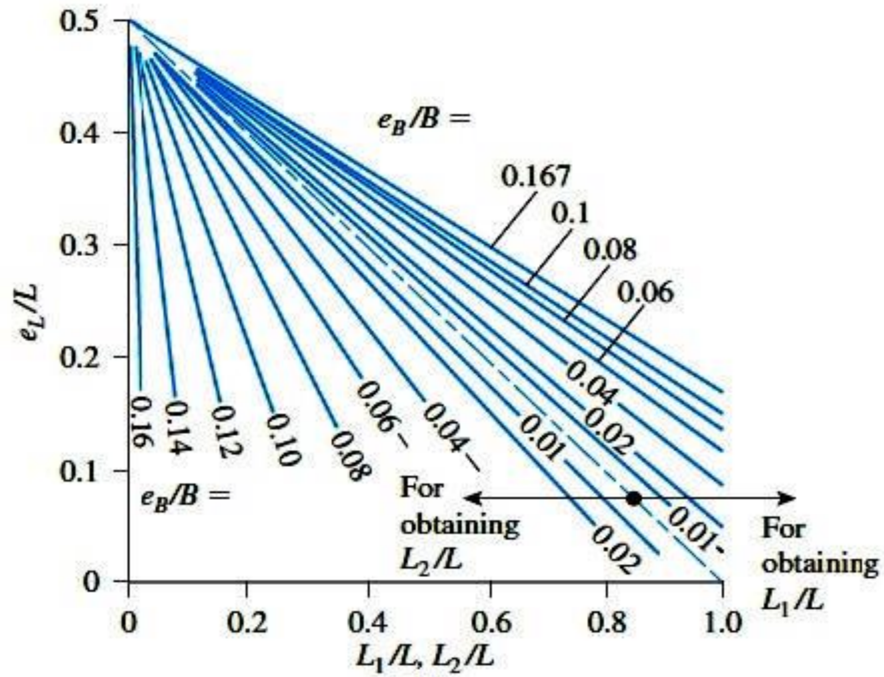
Ultimate Bearing Capacity



(b)

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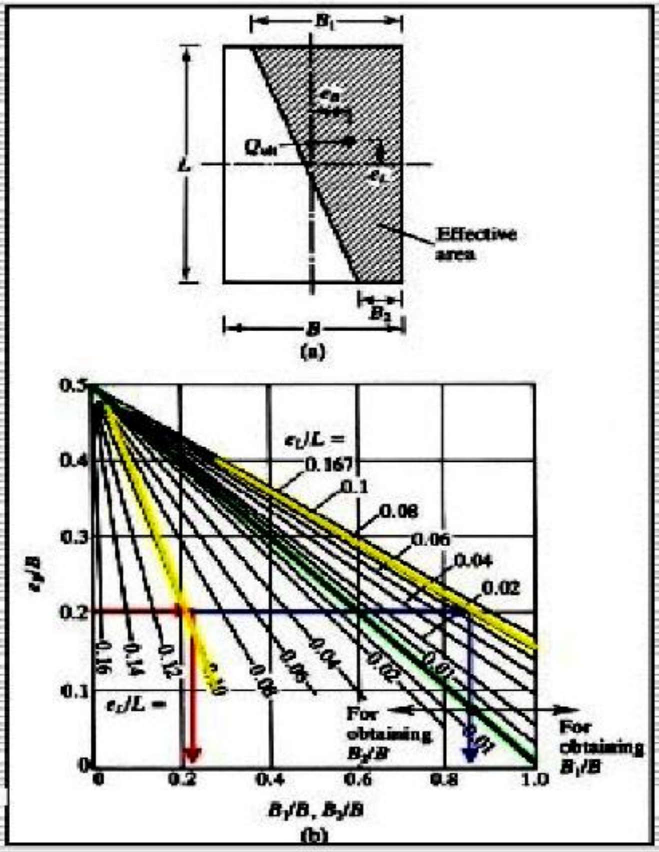
Ultimate Bearing Capacity

- Case 3 $e_L/L < 1/6$ and $0 < e_B/B < 0.5$



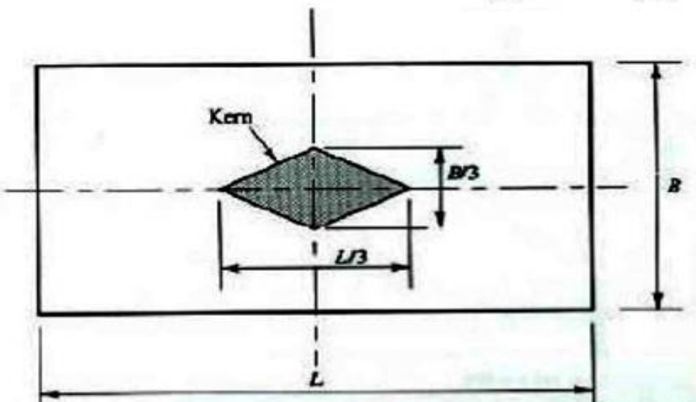
$$A' = \tfrac{1}{2}(B_1 + B_2)L$$
$$B' = A'/L$$
$$L' = L$$

Ultimate Bearing Capacity

$$q = \frac{Q}{A}\left(1 \pm 6\frac{e_B}{B} \pm 6\frac{e_L}{L}\right)$$

For contact pressure to remain (+) ve everywhere,

$$\frac{6e_B}{B} + \frac{6e_L}{L} \leq 1.0$$



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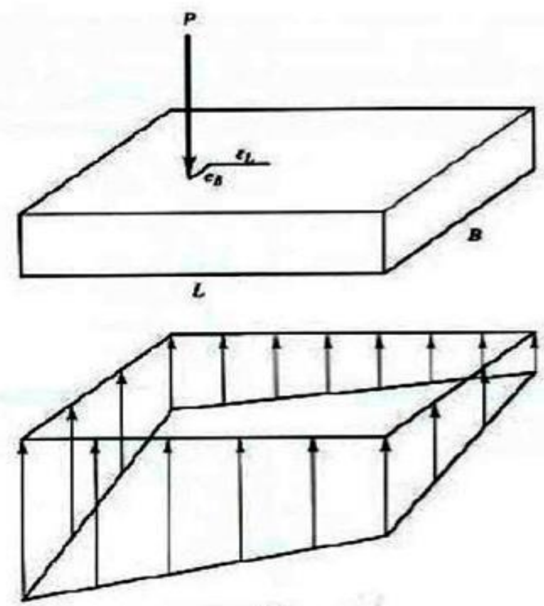

(a)

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Ultimate Bearing Capacity

BEARING CAPACITY FROM SPT

- Several papers have been published that provide statistical data that predict the bearing capacity of footings whilst controlling their settlement to 1 inch. The data is based on the results of SPTs with a correction to 70%, that is N_{70}. The allowable bearing capacity can be provided on a preliminary basis from,

$$q_{all} = \frac{N_{70}}{0.04}\left(1 + 0.33\frac{D_f}{B}\right) \quad if \; B \le 1.2\,m$$

$$q_{all} = \frac{N_{70}}{0.06}\left(\frac{B+0.3}{B}\right)^2\left(1 + 0.33\frac{D_f}{B}\right) \quad if \quad B \ge 1.2\,m$$

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Ultimate Bearing Capacity

Bearing Capacity using CPT

$$q_c \sim 0.8N_q \sim 0.8N_\gamma$$

For Granular Soils:

strip footings $q_{ult} = 28 - 0.0052(300 - q_c)^{1.5} \dfrac{kg}{cm^2}$

square footings $q_{ult} = 48 - 0.009(300 - q_c)^{1.5} \dfrac{kg}{cm^2}$

For Cohesive Soils:

strip footings $q_{ult} = 2 + 0.28 q_c \dfrac{kg}{cm^2}$

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The End